# Electromagnetism and Gravitation: A terrific duality

The Theory of the (E) Question

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## Contents



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# 1 A terrific duality

## 1.1 The dilemna

The GTR2 approach, if it correctly describes some parts of our reality, introduces what I shall call a "terrific duality". What do I mean with these words? Nothing else but: there are situations for which the electromagnetic (EM) fields are the product of variations of the geometrical substrate. This duality opens a fundamental problem of interpretation. What exactly does a relation like:

$$_{\alpha\beta}F_{\lambda\mu}(GTR2)$$

$$=$$

$$-\frac{1}{2}\,.\,(q^{\alpha\beta}\;+\;q^{\beta\alpha})\,.\,R_{\alpha\beta\lambda\mu}(Riemann)\;\equiv\;-2g\,.\,t^{\alpha\beta}\,.\,R_{\alpha\beta\lambda\mu}(Cartan)$$

tells us? Are the unavoidable variations of the geometrical substrate producing a background noise in our universe that we are unable to differentiate from the cosmic microwave background (CMB)? In that new document, I try to illuminate the debate in looking at it under another perspective. I consider alternative ways of thinking that are yielding other visages for the EM fields. And here too, I discover situations where the duality can appear.

## 1.2 A. Einstein versus W. Heisenberg: comments

In section 3(b), I have proposed a scientific but, unconventional and totally politically incorrect, confrontation between A. Einstein's [[01]] and W. Heisenberg's [[02]] masters works.

One of the most chocking issue arising from the discussion developed in my preprint certainly is the fact that the solutions of general relativity (GR) (i.e.: the invariant (ds)$^2$) can now be understood as a logical sub-product of (i) Heisenberg's uncertainty principle (HUP) for the pair (energy, time), and (ii) E.B. Christoffel's work [[03]] focusing on the preservation of differential forms.

$$\begin{array}{ccccc} invariant\,minimum:\;h/4\pi & \longleftarrow & h/4\pi\;\leq\;\delta W.\delta t & \longleftarrow & HUP\,[[02]] \\ \downarrow & & & & \\ Christoffel's\,work\,[03] & \longrightarrow & section3(b) & \longrightarrow & (ds)^2 = invariant \end{array}$$

Figure 1: proposition in section 4(b)

In that sense, it may be affirmed that the ideas which are promoted in [[02]] and [[03]] bring arguments short cutting the way of thinking exposed in [[01]] ... but, arriving at the same fundamental result: the invariance of the (ds)$^2$. That invariance is suggested by the Morley and Michelson experiment [[12]]. The logical progression that I have proposed in section 3(b) is only reorganizing the historical chronology: see figure 2.

The proposition section 3(b) also opens two new roads for the research:

1. in introducing a new fundamental connection preserving the Planck's constant: h.

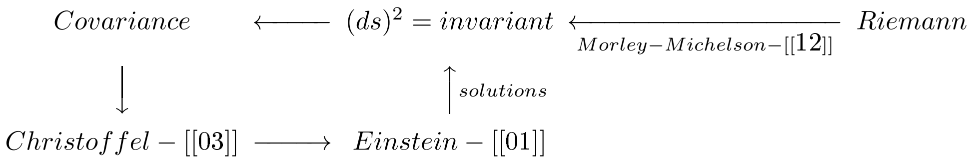


Figure 2: Historical chronology

2. in introducing a new formalism for the electromagnetic (EM) fields. In that formalism, the metric tensor appears explicitly and this fact is an indirect consequence of the extrinsic method (for further explanation, see for example section 3(d)). This method is itself a specific procedure developed within the theory studying the deformed tensor products and their decompositions. That theory is the so-called © Theory of the (E) question (TEQ) which should not be confused with the E-Theory [[04]]. For basic definitions concerning the TEQ, read the technical intermezzo in section 3(a).

## 1.3 Premisses

The Lorentz-Einstein law (LEL) can be written in $E^*(4, R)$:

$$m \,.\, | \frac{du^\alpha}{ds} + \Gamma^\alpha_{\beta\chi} \,.\, u^\beta \,.\, u^\chi > = q \,.\, [F^\alpha{}_\beta] \,.\, |u^\beta > \tag{1}$$

That law is a generalization of the Lorentz law. Its formalism differs from the initial, historical and classical formulation through the so-called gravitational term. The origin of that supplementary term is rooted in considerations developed around the concepts of co-variant derivation and parallel transport. That term is obviously a deformed tensor product which may be written:

$$\otimes_{\Gamma(2)}(\mathbf{p}, \mathbf{u})$$

and, because of that, the law can be interpreted as a natural realization of the TEQ. This fact alone legitimates the treatment of that law with the extrinsic method. I obtain the following generic formulation:

$$q \,.\, [F^\alpha{}_\beta] \;=\; {}_{\Gamma(2)}\Phi(\mathbf{p}) \;-\; [B]^{-1} \,.\, Hess_{(0,\,\mathbf{u})} P_2(\mathbf{u} \;=\; \mathbf{0}) \tag{2}$$

where (i) $\Gamma(2)$ represents the cube which can be formed with Christoffel's symbols of the second kind (the $\Gamma^\alpha{}_{\beta\chi}$), (ii) $\mathbf{u}$: ($u^0$, $u^1$, $u^2$, $u^3$) is the four-speed of the particle at hand and $\mathbf{p}$ its kinetic momentum, (iii) [B] may represent the metric tensor [G] but could also be any non-degenerate bi-linear form acting in a four-dimensional space, (iv) $P_2$ is a polynomial depending on the four-speed and (v) $Hess_{(0,\mathbf{u})}$ must be understood as a symbol representing a Hessian calculated in a flat space (this explains the presence of the "0") in involving partial derivations by respect for the components of $\mathbf{u}$. All along this new document, I shall try to convince the readers that the whole approach is meaningful for fundamental physics and for applied physics as well.

## 1.4 Coming back to the early hours of the theory - a short autobiography

The TEQ has a long history. A short version of the early hours can be discovered in section 3(f). Although that document was involving a minimal kit of mathematical tools and developing really naive demonstrations, it still introduced the basic elements for a deeper and more fundamental discussion (the cross product: as a prototype for the exterior product and for the Lie product; the matrix representing a rotation: as prehistoric illustration for the future trivial decompositions within the TEQ). Its most impressive achievement lies in a demonstration proving the existence of neutral energetic flows in a very classical "Maxwell's vacuum", exactly where nothing should have happened.

It also softly introduces what will become the most important idea of my theory: for some unclear reasons, tensor products may eventually be deformed by a natural context and that idea has immediate consequence in physics; e.g.: in cosmology (again, see section 3(a) and in the conform quantum field theory (CFT) with the concept of flavor exchange as well.

The original version of the document section 3(f) has been written in the middle of the seventies (physics was already my hobby), after a controversial meeting with the unfortunate advocate of the so-called synergistic theory [[05]], just before I decided to start my studies in dentistry (1976-1982). More precisely: at a time where neutral flows (the Zs) had just been predicted by Abdus Salam, Sheldon Glashow and Steven Weinberg (1973)[1] and the masses were not yet known (They only have been available in 1983). Motivated by the positive reactions of some professional working in physics and, in some way forced by a brain stroke which broke my dentist career, I restarted my research in physics in 2004, scrutinizing the mathematics deeper.

The first step in that way has been guided by two obsessions: (a) the rigorous definition of a generalized (deformed) Lie product; (b) the writing of a theory explaining how and why deformed Lie products should be non-trivially decomposed accordingly to local circumstances. The technical details concerning the point (a) can be read in the document section 3(a). The first English version of the quest concerning the point (b) has been developed in a three-dimensional space: it is my so-called *intrinsic method*; see section 3(c).

After the discovery of the intrinsic method, my progression has been stopped during a long time because that method appeared to be a kind of accident, was based on a long demonstration (the "initial theorem") which was difficult to check, could not yield the residual part of a decomposition and was only working in a three-dimensional environment! The situation could slowly change and positively evolve after the discovery of the extrinsic method (section 3(d) working in any dimension! and its confrontation, in a three-dimensional space (section 3(e)), with the non achieved result of section 3(c).

## 1.5 The polymorphic EM fields

Concerning physics, I realized in 2004 that the Lorentz-Einstein law can be written under a differential operator formalism (within a "Sturm-Liouville"-like theory - for the academic presentation see, e.g.: [[06]]). My work is now available in English language in section 3(h). It introduces a connection acting on the (up, down) version of the EM field tensor and that connection can be restricted to the very classical well-known gauge for mass-less particles (e.g.: photons) if one imposes the so-called "golden rule" to the first coefficient (a (4-4) matrix labeled $[{}_0\mathrm{P}]$ which can be identified with the Jacobi's formulation of the Lorentz transformations: $[\Lambda]$) of

[1] But I was not aware of this fact at that time!

that differential formalism:

$$([_0P]^t)^{-1} = ([_0P]^{-1})^t \tag{3}$$

It is a matter of facts that this rule also plays a crucial role in our ability to link the formalism of the EM field which is resulting from the extrinsic method, (02), with the one involved in that classical gauge. For a polynomial $P_2$ without discontinuity in $\mathbf{u} = \mathbf{0}$ (the Hessian is symmetric), it can be demonstrated the:

## 1.6 First remarkable results

**Theorem 1.1.** ***Equivalent representations for the EM fields of the theory***

If, and only if, the golden rule holds true for a non-degenerate bi-linear form [B], $([B]^t)^{-1} = ([B]^{-1})^t$, then the formulation of the EM field tensor given by the extrinsic method - when applied to the Lorentz-Einstein law-, (01), and the following relation are totally equivalent:

$$[B] \,.\, {}_{\Gamma(2)}\Phi(\mathbf{p}) - {}_{\Gamma(2)}\Phi^t(\mathbf{p}) \,.\, [B]^t = [B] \,.\, [F^{\alpha}{}_{\beta}] - [F^{\alpha}{}_{\beta}]^t \,.\, [B]^t$$

**Corollary 1.1.** ***Interpretation: EM fields with a dynamical and geometric origin.***

This new representation may be interpreted as follows: "The existence of a symmetric Hessian in $\mathbf{u} = \mathbf{0}$ and of a non-degenerated bi-linear form [B] respecting the golden rule is defining a kind of equivalence between all non-trivial (up, down) tensor representations of the EM field which are centered on a given trivial decomposition of the gravitational term characterizing the Lorentz-Einstein law: $\otimes_{\Gamma(2)}(\mathbf{p}, \mathbf{u})$".

If, after that, we ask the legitimate question: "What are these expressions representing?", we then get an immediate answer when the bi-linear form is identified with the metric tensor (i.e.: [B] = [G]). Indeed, with the help of the tensor calculus, we rapidly recognize the difference between the (2, 0) tensor representation of the EM field and its transposed; recall that:

$$[F_{\alpha\beta}] = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -H_z & H_y \\ -E_y & H_z & 0 & -H_x \\ -E_z & -H_y & H_x & 0 \end{bmatrix} ; \; [F^{\alpha\beta}] = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{bmatrix}$$

Where $\mathbf{E}$ : ($E_x$, $E_y$, $E_z$) is the electric field and $\mathbf{H}$ : ($H_x$, $H_y$, $H_z$) is the magnetic field. Since the (2, 0) representation is skew-symmetric, this yields:

$$2 \,.\, [F_{\alpha\beta}] = [G] \,.\, {}_{\Gamma(2)}\Phi(\mathbf{p}) - {}_{\Gamma(2)}\Phi^t(\mathbf{p}) \,.\, [G]^t = [G] \,.\, [F^{\alpha}{}_{\beta}] - [F^{\alpha}{}_{\beta}]^t \,.\, [G]^t$$

**Conclusion**: the theory implicitly introduces EM fields centered on a trivial decomposition which has obviously nothing to do with electromagnetism! These hypothetical EM fields are supposed to exist when two conditions are simultaneously realized: (a) the particle at hand has a (relative) speed (b) in a changing geometrical environment.

**Remark 1.1.** ***Criticism and doubts***

Do these fields really exist in the nature? Are they not only ghost fields or the result of some misleading mathematical artifact? And if not, if they really exist, where are they effectively realized? Particles moving in a changing geometry are spontaneously evocative of black holes. Important concentrations of matter rumple the space-time: this is a conclusion inherited from the vision developed by the theory of relativity and this is actually nothing new; but is that rumbling associated with the birth of EM fields as suggested by my theory and by Hawking's work? And aren't they yet quite more cases offering opportunities for the appearance of such fields? For example: what happens when we grate the surface of our bread? At a molecular scale, we may eventually break some chemical connections and deform the geometry at that level. Does this maneuver create superficial EM fields indirectly related to electrostatic phenomenon? I don't know but I can imagine it, although it is difficult to define the concept of surface precisely when we consider a unique molecule.

## 1.7 The exact formalism of these EM fields

**Lemma 1.1.** ***Electromagnetism and variations of the metric***

Anyway, provided that category of EM fields described with (03) effectively exists, as I have extensively explained it in section 3(i), if:

$$(04-a)\ :\ {}_{\Gamma(2)}\Phi(\mathbf{p}) = \begin{pmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{pmatrix};\ \Upsilon = \pm\chi = {\Gamma^0}_{\mu 3} \,.\, p^\mu;\ [G] = [G]^t$$

then, due to the work done in [[07]; §§172-173], there exists a sub-category such that:

$$(04-b)\ :\ 2\,.\,[F_{\alpha\beta}] = [G]\,.\,[{F^\alpha}_\beta] - [{F^\alpha}_\beta]^t\,.\,[G] = [G]\,.\,{}_{\Gamma(2)}\Phi(\mathbf{p}) - {}_{\Gamma(2)}\Phi(\mathbf{p})\,.\,[G] = \delta[G]$$

None of the elements in that sub-category can be distinguished from an infinitesimal variation of the geometry! Explanation: the left hand part of (04-b) is only an illustration of theorem 1.1 and the latter is obtained in applying the extrinsic method applied to the LEL (03). The right hand part of (04-b) is a direct consequence of the work done and exposed in [[07]; §§172-173]. That term is the result of any rotation acting on the metric (understood as a vector in M(4, $\mathbb{R}$ or $\mathbb{C}$)). The work realized in section 3(i) is just my analysis/illustration of [[07]] when the rotation is represented by a trivial decomposition inherited from the Lorentz-Einstein law (LEL). "*But do EM fields with that strange property really exist in the nature?*"

**Corollary 1.2.** ***Interpretation***

Let write the exact formalism of these fields.

$$\forall\,[G] = [G]^t\ :\ [G]\,.\,{}_{\Gamma(2)}\Phi(\mathbf{p})$$

$$=$$

$$\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \chi\cdot g_{03} & \Upsilon\cdot g_{02} & \Upsilon\cdot g_{01} & \chi\cdot g_{00} \\ \chi\cdot g_{13} & \Upsilon\cdot g_{12} & \Upsilon\cdot g_{11} & \chi\cdot g_{01} \\ \chi\cdot g_{23} & \Upsilon\cdot g_{22} & \Upsilon\cdot g_{12} & \chi\cdot g_{02} \\ \chi\cdot g_{33} & \Upsilon\cdot g_{23} & \Upsilon\cdot g_{13} & \chi\cdot g_{03} \end{bmatrix}$$

And:

$$\forall [G] = [G]^t : {}_{\Gamma(2)}\Phi(\mathbf{p}) . [G]$$

$$=$$

$$\begin{bmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \chi \cdot g_{03} & \chi \cdot g_{13} & \chi \cdot g_{23} & \chi \cdot g_{33} \\ \Upsilon . g_{02} & \Upsilon . g_{12} & \Upsilon . g_{22} & \Upsilon . g_{23} \\ \Upsilon . g_{01} & \Upsilon . g_{11} & \Upsilon . g_{12} & \Upsilon . g_{13} \\ \chi \cdot g_{00} & \chi \cdot g_{01} & \chi \cdot g_{02} & \chi \cdot g_{03} \end{bmatrix}$$

This is resulting in:

$$\forall [G] = [G]^t : [G] . {}_{\Gamma(2)}\Phi(\mathbf{p}) - {}_{\Gamma(2)}\Phi(\mathbf{p}) . [G]$$

$$=$$

$$\begin{bmatrix} 0 & \Upsilon . g_{02} - \chi . g_{13} & \Upsilon . g_{01} - \chi . g_{23} & \chi . (g_{00} - g_{33}) \\ \chi \cdot g_{13} - \Upsilon . g_{02} & 0 & \Upsilon . (g_{11} - g_{22}) & \chi \cdot g_{01} - \Upsilon . g_{23} \\ \chi \cdot g_{23} - \Upsilon . g_{01} & \Upsilon . (g_{22} - g_{11}) & 0 & \chi \cdot g_{02} - \Upsilon . g_{13} \\ \chi \cdot (g_{33} - g_{00}) & \Upsilon . g_{23} - \chi . g_{01} & \Upsilon . g_{13} - \chi . g_{02} & 0 \end{bmatrix}$$

As expected, that formalism is mimicking a F(2, 0) representation if we can write:

$$E_x = \Upsilon . g_{02} - \chi . g_{13};\ E_y = \Upsilon . g_{01} - \chi . g_{23};\ E_z = \chi . (g_{00} - g_{33});$$

$$H_x = \Upsilon . g_{13} - \chi . g_{02};\ H_y = \chi . g_{01} - \Upsilon . g_{23};\ H_z = \Upsilon . (g_{22} - g_{11});$$

But in fact, we now face a logical problem which is annihilating our hopes. Indeed, since that matrix is also supposed to be the representation of the infinitesimal variation of a symmetric metric tensor, we have to write:

$$\delta g_{\alpha\beta} = \delta g_{\beta\alpha}$$

and, for coherence, that new EM field must vanish; at the end of the day, we stay with a deception, no variation of the geometry and the following constraints:

$$E_x = \Upsilon . g_{02} - \chi . g_{13} = E_y = \Upsilon . g_{01} - \chi . g_{23} = E_z = \chi . (g_{00} - g_{33}) = 0$$

$$H_x = \Upsilon . g_{13} - \chi . g_{02} = H_y = \chi . g_{01} - \Upsilon . g_{23} = H_z = \Upsilon . (g_{22} - g_{11}) = 0$$

The dependence on the variations of the geometry and on the kinetic momentum can always be eliminated; effectively if:

$$\forall \chi = \Upsilon = {\Gamma^0}_{\mu 3} . p^\mu$$

$$\Downarrow$$

$$g_{02} - g_{13} = g_{01} - g_{23} = g_{00} - g_{33} = 0$$

$$g_{13} - g_{02} = g_{01} - g_{23} = g_{22} - g_{11} = 0$$

$$\Downarrow$$

$$[G] = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{02} \\ g_{02} & g_{12} & g_{11} & g_{01} \\ g_{03} & g_{02} & g_{01} & g_{00} \end{bmatrix}$$

And if:

$$\forall \chi = -\Upsilon = \Gamma^0{}_{\mu 3} \cdot p^\mu$$

$$\Downarrow$$

$$-g_{02} - g_{13} = -g_{01} - g_{23} = g_{00} - g_{33} = 0$$

$$-g_{13} - g_{02} = g_{01} + g_{23} = g_{11} - g_{22} = 0$$

$$\Downarrow$$

$$[G] = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & -g_{02} \\ g_{02} & g_{12} & g_{11} & -g_{01} \\ g_{03} & -g_{02} & -g_{01} & g_{00} \end{bmatrix}$$

**Conclusion of corollary 1.2**: With the interpretation proposed in [[07]; §§172-173] for the formalism (03) when (04-a) holds true, a trivial decomposition representing the Lorentz-Einstein law has no action on two sub-sets of metrics (i.e.: they stay unchanged; synonym: they are invariant under the action of this trivial decomposition).

## 1.8 Can we extend the formalism?

In limiting the discussion to the vision exposed in [[07]], the lemma 1.1 may appear to be a kind of "*no-go theorem*" and I should stop the exploration here. In fact, the situation is a little bit more subtle because we are in front of two predictions/relations arising from two different viewpoints. Let for example now try a similar calculation than previously, but with any metric:

$$\forall [G] \,:\, [G] \,.\, {}_{\Gamma(2)}\Phi(\mathbf{p})$$

$$=$$

$$\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \chi \cdot g_{03} & \Upsilon \cdot g_{02} & \Upsilon \cdot g_{01} & \chi \cdot g_{00} \\ \chi \cdot g_{13} & \Upsilon \cdot g_{12} & \Upsilon \cdot g_{11} & \chi \cdot g_{10} \\ \chi \cdot g_{23} & \Upsilon \cdot g_{22} & \Upsilon \cdot g_{21} & \chi \cdot g_{20} \\ \chi \cdot g_{33} & \Upsilon \cdot g_{32} & \Upsilon \cdot g_{31} & \chi \cdot g_{30} \end{bmatrix}$$

And:

$$\forall [G] \,:\, {}_{\Gamma(2)}\Phi(\mathbf{p}) \,.\, [G]^t$$

$$=$$

$$\begin{bmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} g_{00} & g_{10} & g_{20} & g_{30} \\ g_{01} & g_{11} & g_{21} & g_{31} \\ g_{02} & g_{12} & g_{22} & g_{32} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \chi \cdot g_{03} & \chi \cdot g_{13} & \chi \cdot g_{23} & \chi \cdot g_{33} \\ \Upsilon \cdot g_{02} & \Upsilon \cdot g_{12} & \Upsilon \cdot g_{22} & \Upsilon \cdot g_{32} \\ \Upsilon \cdot g_{01} & \Upsilon \cdot g_{11} & \Upsilon \cdot g_{21} & \Upsilon \cdot g_{31} \\ \chi \cdot g_{00} & \chi \cdot g_{10} & \chi \cdot g_{20} & \chi \cdot g_{30} \end{bmatrix}$$

This is now resulting in:

$$\forall\,[G]\,:\,[G]\,.\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,-\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,.\,[G]^t$$

$$=$$

$$\begin{bmatrix} 0 & \Upsilon\,.\,g_{02}\,-\,\chi\cdot g_{13} & \Upsilon\,.\,g_{01}\,-\,\chi\cdot g_{23} & \chi\,.\,(g_{00}\,-\,g_{33}) \\ \chi\cdot g_{13}\,-\,\Upsilon\,.\,g_{02} & 0 & \Upsilon\,.\,(g_{11}\,-\,g_{22}) & \chi\cdot g_{10}\,-\,\Upsilon\,.\,g_{32} \\ \chi\cdot g_{23}\,-\,\Upsilon\,.\,g_{01} & \Upsilon\,.\,(g_{22}\,-\,g_{11}) & 0 & \chi\cdot g_{20}\,-\,\Upsilon\,.\,g_{31} \\ \chi\,.\,(g_{33}\,-\,g_{00}) & \Upsilon\,.\,g_{32}\,-\,\chi\cdot g_{10} & \Upsilon\,.\,g_{31}\,-\,\chi\cdot g_{20} & 0 \end{bmatrix}$$

As expected and predicted by the extrinsic method plus the corollary of theorem 1.1, that formalism is once again mimicking a F(2, 0) representation if we can write:

$$E_x\,=\,\Upsilon\,.\,g_{02}\,-\,\chi\,.\,g_{13};\;E_y\,=\,\Upsilon\,.\,g_{01}\,-\,\chi\,.\,g_{23};\;E_z\,=\,\chi\,.\,(g_{00}\,-\,g_{33});$$

$$H_x\,=\,\Upsilon\,.\,g_{31}\,-\,\chi\,.\,g_{20};\;H_y\,=\,\chi\,.\,g_{01}\,-\,\Upsilon\,.\,g_{32};\;H_z\,=\,\Upsilon\,.\,(g_{22}\,-\,g_{11});$$

But/and here, *that expression cannot be confronted with [[07]; §§172-173, pp.145-147]* because, despite of the obvious symmetry of the trivial decomposition at hand, the calculations have been made with the transposed of the metric. Any way, the theorem 1.1. holds true and we can write again (but in a simplified manner):

$$F(2,0)\,=\,G\,.\,\Phi\,-\,\Phi^t\,.\,G^t;\;\Phi\,=\,\Phi^t$$

So, let now calculate the expression needed by an approach which would be guided by [[07]]:

$$\forall\,[G]\,:\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,.\,[G]$$

$$=$$

$$\begin{bmatrix} 0 & 0 & 0 & \chi \\ 0 & 0 & \Upsilon & 0 \\ 0 & \Upsilon & 0 & 0 \\ \chi & 0 & 0 & 0 \end{bmatrix}\,.\,\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} \chi\cdot g_{30} & \chi\cdot g_{31} & \chi\cdot g_{32} & \chi\cdot g_{33} \\ \Upsilon\,.\,g_{20} & \Upsilon\,.\,g_{21} & \Upsilon\,.\,g_{22} & \Upsilon\,.\,g_{23} \\ \Upsilon\,.\,g_{10} & \Upsilon\,.\,g_{11} & \Upsilon\,.\,g_{12} & \Upsilon\,.\,g_{13} \\ \chi\cdot g_{00} & \chi\cdot g_{01} & \chi\cdot g_{02} & \chi\cdot g_{03} \end{bmatrix}$$

This is resulting in:

$$\forall\,[G]\,:\,[G]\,.\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,-\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,.\,[G]$$

$$=$$

$$\begin{bmatrix} \chi\,.\,(g_{03}\,-\,g_{30}) & \Upsilon\,.\,g_{02}\,-\,\chi\cdot g_{31} & \Upsilon\,.\,g_{01}\,-\,\chi\cdot g_{32} & \chi\,.\,(g_{00}\,-\,g_{33}) \\ \chi\cdot g_{13}\,-\,\Upsilon\,.\,g_{20} & \Upsilon\,.\,(g_{12}\,-\,g_{21}) & \Upsilon\,.\,(g_{11}\,-\,g_{22}) & \chi\cdot g_{10}\,-\,\Upsilon\,.\,g_{23} \\ \chi\cdot g_{23}\,-\,\Upsilon\,.\,g_{10} & \Upsilon\,.\,(g_{22}\,-\,g_{11}) & \Upsilon\,.\,(g_{21}\,-\,g_{12}) & \chi\cdot g_{20}\,-\,\Upsilon\,.\,g_{13} \\ \chi\,.\,(g_{33}\,-\,g_{00}) & \Upsilon\,.\,g_{32}\,-\,\chi\cdot g_{01} & \Upsilon\,.\,g_{31}\,-\,\chi\cdot g_{02} & \chi\,.\,(g_{30}\,-\,g_{03}) \end{bmatrix}$$

Except if both, the trivial decomposition and the metric, are represented by symmetric matrices, this formalism can in general no more be interpreted with the corollary of theorem 1.1 (see above) but it is always understandable with the help of [[07]; §§172-173, pp.145-147] as an infinitesimal variation of the metric ( up to a minus sign) when the trivial decomposition at hand represents a rotation. Since it is a matter of facts that, in Dyson's semantic [[08]; p. 30], the trivial decomposition at hand, (04-b), is proportional to one of the Dirac's matrices:

$$(05)\,:\,\chi\,=\,\Upsilon\,\Rightarrow\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,=\,\chi\,.\,\alpha^1;\;\chi\,=\,-\Upsilon\,\Rightarrow\,{}_{\Gamma(2)}\Phi(\mathbf{p})\,=\,-\chi\,.\,\gamma^2$$

it is obvious that the considerations developed in [[07]], and after that in section 3(i), hold true here; more precisely, if $\chi = \Upsilon$:

$$-\delta g_{00} = \chi . (g_{03} - g_{30}) = \delta g_{33}$$

$$-\delta g_{01} = \chi . (g_{02} - g_{31}) = \delta g_{32}$$

$$-\delta g_{02} = \chi . (g_{01} - g_{32}) = \delta g_{31}$$

$$-\delta g_{03} = \chi . (g_{00} - g_{33}) = \delta g_{30}$$

$$-\delta g_{10} = \chi . (g_{13} - g_{20}) = \delta g_{23}$$

$$-\delta g_{11} = \chi . (g_{12} - g_{21}) = \delta g_{22}$$

$$-\delta g_{12} = \chi . (g_{11} - g_{22}) = \delta g_{21}$$

$$-\delta g_{13} = \chi . (g_{10} - g_{23}) = \delta g_{20}$$

And if $\chi$ = -$\Upsilon$:

$$-\delta g_{00} = \chi . (g_{03} - g_{30}) = \delta g_{33}$$

$$-\delta g_{01} = -\chi . (g_{02} + g_{31}) = \delta g_{32}$$

$$-\delta g_{02} = -\chi . (g_{01} + g_{32}) = \delta g_{31}$$

$$-\delta g_{03} = \chi . (g_{00} - g_{33}) = \delta g_{30}$$

$$-\delta g_{10} = \chi . (g_{13} + g_{20}) = \delta g_{23}$$

$$-\delta g_{11} = -\chi . (g_{12} - g_{21}) = \delta g_{22}$$

$$-\delta g_{12} = -\chi . (g_{11} - g_{22}) = \delta g_{21}$$

$$-\delta g_{20} = \chi . (g_{23} + g_{10}) = \delta g_{13}$$

*In all cases the infinitesimal variation of the metric contains at most eight independent coefficients.*

$$(06 - a) :$$

$$\forall [G] : [G] . {}_{\Gamma(2)}\Phi(\mathbf{p}) - {}_{\Gamma(2)}\Phi(\mathbf{p}) . [G] = \delta[G] = \begin{bmatrix} \delta g_{00} & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ \delta g_{10} & \delta g_{11} & \delta g_{12} & -\delta g_{20} \\ \delta g_{20} & -\delta g_{12} & -\delta g_{11} & -\delta g_{10} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & -\delta g_{00} \end{bmatrix}$$

*This formula must be re-situated into the context of [[07]; §172, p. 145]*; concretely, it is only a part of a more complete one explaining how the metric (understood as a vector of M(4, R or C)) changes under the action of a rotation (more precisely: a double reflection) represented by the trivial decomposition at hand:

$$(06 - b) : \forall [G] : [G] \to [G'] = [G] + \delta[G]$$

In our case, the initial metric is transformed into:

$$\forall [G] : [G] \to [G'] = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} + \begin{bmatrix} \delta g_{00} & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ \delta g_{10} & \delta g_{11} & \delta g_{12} & -\delta g_{20} \\ \delta g_{20} & -\delta g_{12} & -\delta g_{11} & -\delta g_{10} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & -\delta g_{00} \end{bmatrix}$$

Any way, as claimed at the beginning of that paragraph, we have now two generic relations (where, for simplicity, $\Phi$ denotes a rotation); one arising from the extrinsic method:

$$F(2,0) = [G] . \Phi - \Phi^t . [G]^t ; \Phi = \Phi^t$$

and another one arising from [[07]]:

$$\delta[G] = [G] \, . \, \Phi - \Phi \, . \, [G];$$

The confrontation yields in general:

$$[G] \, . \, \Phi = F(2, 0) + \Phi^t \, . \, [G]^t = \delta[G] + \Phi \, . \, [G]$$

This is giving a part of the answer to my initial question concerning the *terrific duality* (see subsection 1.1); namely: "Do infinitesimal variations of the metric *generate* EM fields?"

**Theorem 1.2.** ***Electromagnetism and variation of the geometry***

In general, for any metric represented in M(4, ℝ) by a matrix [G] and for any rotation represented in M(4, ℝ) by a matrix Φ, the confrontation between both viewpoints ([[07]] and document section 3(i)) yields:

$$(07) \, : \, \delta[G] = F(2,0) + (\Phi^t \, . \, G^t - \Phi \, . \, G)$$

The relation (07) contains diverse situations which I shall describe progressively, for example with a table like:

| | [G] = $[G]^t$ | [G] = -$[G]^t$ |
|---|---|---|
| $_A\Phi = {_A\Phi^t}$ | δ[G] = F(2,0) = [0] | |
| $_A\Phi$ = -$_A\Phi^t$ | | |

That theorem reveals why it is crucial to know all possible representations of rotations in a four dimensional space. This necessary work as been started in [[07]] and has actually important prolongations, for example in [[09]].

**Example 1.1.** ***Recovering the limit case of lemma 1.1***

When the trivial decomposition (the rotation) is represented by a symmetric matrix, then (07) yields:

$$\delta[G] = F(2,0) + \Phi \, . \, ([G]^t - [G])$$

If, furthermore, the metric is symmetric, then the infinitesimal variation of the metric is an EM field but the "no-go corollary" of lemma 1.1 imposes the vanishing of that EM field.

**Example 1.2.** ***A non-symmetric metric of which the symmetric part is preserved***

At a first glance, there is no reason why the infinitesimal variation of the metric should not sometimes generate an EM field when the trivial decomposition is symmetric but not the metric; or conversely (the trivial decomposition is not symmetric and the metric is symmetric).

For example here, if $\chi$ = -Υ (see (05) above) and if the metric is not symmetric ([G] ≠ $[G]^t$):

$$\forall [G] \, : \, [G] \to [G'] = [G] + F(2,0) - \chi \, . \, \gamma^2 \, . \, ([G]^t - [G])$$

where $\gamma^2$ is not a square but one of the Dirac's matrices. In that case:

$$\forall [G] \, : \, [G] \to [G']^t = [G]^t - F(2,0) - \chi \, . \, \gamma^2 \, . \, ([G] - [G]^t)$$

A first consequence of that kind of transformations is that it preserves the symmetric part of the initial metric; indeed, in adding the two previous relations:

$$\forall [G] \, : \, [G] \to [G'] \, : \, [G'] + [G']^t = [G] + [G]^t$$

## 1.9 Metrics of which the variations always produce an EM-like field

**Definition 1.1.** ***EM-like field***

A EM-like field is a variation of the metric which is mimicking the formalism of a representation of a EM-field tensor.

**Remark 1.2.** ***EM-like field - theoretical existence***

Let come back to the expression (06-a) and visualize how it could be separated into two parts, one of them having the formalism of some EM field. Remark that if one imposes only two constraints in the initial metric; precisely:

$$g_{01} = -g_{10},\ g_{02} = -g_{20},$$

then:

$$[G]$$
$$\downarrow$$
$$[G']$$
$$=$$
$$\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} + \begin{bmatrix} \delta g_{00} & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ \delta g_{10} & \delta g_{11} & \delta g_{12} & -\delta g_{20} \\ \delta g_{20} & -\delta g_{12} & -\delta g_{11} & -\delta g_{10} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & -\delta g_{00} \end{bmatrix}$$
$$=$$
$$\begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ -g_{01} & g_{11} & g_{12} & g_{13} \\ -g_{02} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} + \begin{bmatrix} \delta g_{00} & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ -\delta g_{01} & \delta g_{11} & \delta g_{12} & \delta g_{02} \\ -\delta g_{02} & -\delta g_{12} & -\delta g_{11} & \delta g_{01} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & -\delta g_{00} \end{bmatrix}$$
$$=$$
$$\begin{bmatrix} g_{00} + \delta g_{00} & g_{01} & g_{02} & g_{03} \\ -g_{01} & g_{11} + \delta g_{11} & g_{12} & g_{13} \\ -g_{02} & g_{21} & g_{22} - \delta g_{11} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} - \delta g_{00} \end{bmatrix}$$
$$+$$
$$\begin{bmatrix} 0 & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ -\delta g_{01} & 0 & \delta g_{12} & \delta g_{02} \\ -\delta g_{02} & -\delta g_{12} & 0 & \delta g_{01} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & 0 \end{bmatrix}$$

Amazingly, the first part of the r.h.t. is a minimal modification of the initial metric and its trace stays invariant; the second part of the r.h.t has the formalism of a (2, 0) representation of a EM field if it is possible to write:

$$E_x = \delta g_{01} = -H_x;\ E_y = \delta g_{02} = H_y;\ E_z = \delta g_{03};\ H_z = -\delta g_{12}$$

This is suggesting to start the same calculations again, but with different initial conditions for the initial metric:

$$(08-a)\ :\ g_{01} = -g_{10},\ g_{02} = -g_{20},\ g_{03} = g_{30},\ g_{12} = g_{21}$$

In that case, the initial metric is:

$$[G_0] = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ -g_{01} & g_{11} & g_{12} & g_{13} \\ -g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{31} & g_{32} & g_{33} \end{bmatrix}$$

and, whatever the relation between $\chi$ and $\Upsilon$ (i.e.: $\pm$) is, an action of the $\Phi$ matrix described with (04-a) on that kind of metric yields (see above after (05)):

$$\delta g_{00} = \delta g_{11} = 0$$

The great advantage of which being that it leaves this type of metric invariant in producing a EM-like field (i.e.: a variation of the metric which is mimicking the formalism of a representation of a EM-field tensor).

$$(08-b) : [G_0] \to [G'_0] = [G_0] + \begin{bmatrix} 0 & \delta g_{01} & \delta g_{02} & \delta g_{03} \\ -\delta g_{01} & 0 & \delta g_{12} & \delta g_{02} \\ -\delta g_{02} & -\delta g_{12} & 0 & \delta g_{01} \\ -\delta g_{03} & -\delta g_{02} & -\delta g_{01} & 0 \end{bmatrix}$$

The induced EM field would be such that:

$$(08-c) : E_x = \delta g_{01} = -H_x;\ E_y = \delta g_{02} = H_y;\ E_z = \delta g_{03};\ H_z = -\delta g_{12}$$

**Conclusion**: Provided the nature offers circumstances for which, in a region of our space-time, such an action exists, this region should naturally produce EM-like fields of which the real origin and the real nature are in fact geometrical.

This induces a crucial question: “How can an observer makes the distinction between a classical EM field an a EM-like field? In peculiar: what kind of link is there between these EM-like predicted fields and the Thirring-Lense effect [[10]-a, b, c, d-chapter 30]?” See a part of the answer below, in subsection 1.11.

## 1.10 Gravitons

**Remark 1.3.** ***Gravitons and EM-like fields***

In adding only one more condition ($g_{22}$ = - $g_{11}$), the relations (08-a) hold formally true and in peculiar for the case of a graviton; the initial metric is[2]:

$$(09-a) : [H_0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & g_{11} & g_{12} & 0 \\ 0 & g_{12} & g_{22} = -g_{11} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

More precisely, if $\chi = \Upsilon$:

$$-\delta g_{00} = \chi . (g_{03} - g_{30}) = \delta g_{33} = 0$$

$$-\delta g_{01} = \chi . (g_{02} - g_{31}) = \delta g_{32} = 0$$

$$-\delta g_{02} = \chi . (g_{01} - g_{32}) = \delta g_{31} = 0$$

[2]In fact and more precisely, the concept of graviton is commonly understood as the logical result of a specific mathematical treatment applied to Einstein's fields equations. A consequence of which being here that $g_{11}$ and $g_{12}$ must be interpreted as the two components representing the tiny modification of the underlying metric -in extenso: the graviton; not as the underlying metric itself. See for illustration any good book; e.g.: [[11]].

$$-\delta g_{03} = \chi . (g_{00} - g_{33}) = \delta g_{30} = 0$$
$$-\delta g_{10} = \chi . (g_{13} - g_{20}) = \delta g_{23} = 0$$
$$-\delta g_{11} = \chi . (g_{12} - g_{21}) = \delta g_{22} = 0$$
$$-\delta g_{12} = \chi . (g_{11} - g_{22}) = \delta g_{21} = 2.\chi . g_{11}$$
$$-\delta g_{13} = \chi . (g_{10} - g_{23}) = \delta g_{20} = 0$$

And if $\chi$ = -ϒ:

$$-\delta g_{00} = \chi . (g_{03} - g_{30}) = \delta g_{33} = 0$$
$$-\delta g_{01} = -\chi . (g_{02} + g_{31}) = \delta g_{32} = 0$$
$$-\delta g_{02} = -\chi . (g_{01} + g_{32}) = \delta g_{31} = 0$$
$$-\delta g_{03} = \chi . (g_{00} - g_{33}) = \delta g_{30} = 0$$
$$-\delta g_{10} = \chi . (g_{13} + g_{20}) = \delta g_{23} = 0$$
$$-\delta g_{11} = -\chi . (g_{12} - g_{21}) = \delta g_{22} = 0$$
$$-\delta g_{12} = -\chi . (g_{11} - g_{22}) = \delta g_{21} = -2.\chi . g_{11}$$
$$-\delta g_{20} = \chi . (g_{23} + g_{10}) = \delta g_{13} = 0$$

The consequence of which being that the relation (08-b) is now written:

$$(09-b) \,:\, [H_0] \rightarrow [H_0'] = [H_0] + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \pm 2.\chi . g_{11} & 0 \\ 0 & \mp 2.\chi . g_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the EM-like field is reduced to a magnetic field with one component along the z-axis:

$$(09-c) \,:\, E_x = \delta g_{01} = -H_x = 0;\ E_y = \delta g_{02} = H_y = 0;\ E_z = \delta g_{03}$$
$$H_z = -\delta g_{12} = \pm 2.\chi . g_{11}$$

**Conclusion**: This is a remarkable result telling that any gravitational plane wave which is modified by the action of a rotation of that plane, then emits a magnetic signal in a direction orthogonal to that plane.

## 1.11 Thirring-Lense effect

At the end of sub-section 1.10, I have asked if an observer can effectively decide when he/she is in front of a Thirring-Lense effect [10] or facing a EM-like field predicted by the TEQ? Starting with [[10]-d, chapter 30, (30.12 and 14)] the metric of our spinning Earth can be approximately written:

$$(10-a) \,:\, [G_{T-L}]$$
$$=$$
$$\begin{bmatrix} (1 - \frac{2.G.M}{c^2 . r}) . c^2 & \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_1 & \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_2 & \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_3 \\ \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_1 & (1 + \frac{2.G.M}{c^2 . r}) & 0 & 0 \\ \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_2 & 0 & (1 + \frac{2.G.M}{c^2 . r}) & 0 \\ \frac{8.G.M . R_E^2}{5.c^2 . r^3}) . (\omega \wedge \mathbf{r})_3 & 0 & 0 & (1 + \frac{2.G.M}{c^2 . r}) \end{bmatrix}$$

At this stage, *and for the pedagogy*, I would like to insist on the mathematical difficulties encountered by Thirring and Lense in calculating this metric with exactitude. It may be controlled that different results have been obtained and proposed by these authors in 1918 [[10]-c; p. 719, (16)] and in 1921 [[10]-c; p. 726, (16)], [[10]-c; p. 730, (10)]. Furthermore, at a first glance, their last result [[10]-c; p. 730, (10)] is also different from the one of which the demonstration is exposed in [[10]-d]. But, if it is paid attention to the evolution concerning the numeration of subscripts between 1921 and today (the time component was labeled with a "4" instead of with a "0"), everything becomes absolutely fine; especially in considering the implicit simplifying choice $\omega$: $(0, 0, \omega)$ which has been made by the authors and in [[10]-d; chapter 30, p. 164, (30.4)] as well.

Recall that here, $R_E$ is the radius of the Earth, $\mathbf{r}$ is the vector indicating the position which is observed by the observer, r is the amplitude of that vector (and per hypothesis $r \geq R_E$), M is the mass of the Earth, G is the universal constant 6,67. $10^{-11}$ in MKSA units, $\omega = 2\pi/\text{day}$ and $c = 3.\ 10^8$ meters/second is the speed of the light in vacuum.

- Let now come back to (08-a) and state that only two of the four necessary conditions are spontaneously realized; except for the positions where the observer gets the sensation that the first and the second components of $\omega \wedge \mathbf{r}$ vanish. With the choice $\omega$: $(0, 0, \omega)$, this can only be realized when $\mathbf{r}$ is parallel to $\omega$; equivalently: when the Earth is observed from any position situated along its axis of rotation. For these positions, the metric appears to be diagonal and symmetric:

$$(11 - a) \, : \, [G_{T-L}]$$

$$=$$

$$\begin{bmatrix} (1 - \frac{2.G.M}{c^2 \, . \, r}) \, . \, c^2 & 0 & 0 & 0 \\ 0 & (1 + \frac{2.G.M}{c^2 \, . \, r}) & 0 & 0 \\ 0 & 0 & (1 + \frac{2.G.M}{c^2 \, . \, r}) & 0 \\ 0 & 0 & 0 & (1 + \frac{2.G.M}{c^2 \, . \, r}) \end{bmatrix}$$

With a different choice and in general for $\omega$: $(\omega_x, \omega_y, \omega_z)$, since it is supposed that $r \geq R_E$ (i.e.: $r \neq 0$), the two first necessary conditions imposed by (08-a) are:

$$\omega_y \, . \, z \, - \, \omega_z \, . \, y \, = \, 0; \, \omega_z \, . \, x \, - \, \omega_x \, . \, z \, = \, 0; \, \forall \, (\omega \, \wedge \, \mathbf{r})_3 \, = \, \omega_x \, . \, y \, - \, \omega_y \, . \, x$$

But they are imposing:

$$\frac{\omega_x}{x} \, = \, \frac{\omega_y}{y} \, = \, \frac{\omega_z}{z} \, \Rightarrow \, (\omega \, \wedge \, \mathbf{r})_3 \, = \, \omega_x \, . \, y \, - \, \omega_y \, . \, x \, = \, 0$$

and the situation is the same than for the simplified choice. Concretely, the axis of rotation of the Earth is a set of positions where an observer applying the results of the TEQ would affirm (i) that the metric induced by the rotation of the Earth is invariant under the action of the $\Phi$ matrix (04-a) and (ii) that that action is generating a EM-like field. Effectively, if $\chi = \Upsilon$:

$$-\delta g_{00} \, = \, \chi \, . \, (g_{03} \, - \, g_{30}) \, = \, \delta g_{33} \, = \, 0$$

$$-\delta g_{01} \, = \, \chi \, . \, (g_{02} \, - \, g_{31}) \, = \, \delta g_{32} \, = \, 0$$

$$-\delta g_{02} \, = \, \chi \, . \, (g_{01} \, - \, g_{32}) \, = \, \delta g_{31} \, = \, 0$$

$$-\delta g_{03} \, = \, \chi \, . \, (g_{00} \, - \, g_{33}) \, = \, \delta g_{30} \, = \, \chi \, . \, \{c^2 \, . \, (1 \, - \, \frac{2.G.M}{c^2 \, . \, r} \, - \, \frac{2.G.M}{c^4 \, . \, r}) \, - \, 1\}$$

$$-\delta g_{10} \, = \, \chi \, . \, (g_{13} \, - \, g_{20}) \, = \, \delta g_{23} \, = \, 0$$

$$-\delta g_{11} \, = \, \chi \, . \, (g_{12} \, - \, g_{21}) \, = \, \delta g_{22} \, = \, 0$$

$$-\delta g_{12} = \chi.(g_{11} - g_{22}) = \delta g_{21} = 0$$
$$-\delta g_{13} = \chi.(g_{10} - g_{23}) = \delta g_{20} = 0$$

And if $\chi$ = -$\Upsilon$:

$$-\delta g_{00} = \chi.(g_{03} - g_{30}) = \delta g_{33} = 0$$
$$-\delta g_{01} = -\chi.(g_{02} + g_{31}) = \delta g_{32} = 0$$
$$-\delta g_{02} = -\chi.(g_{01} + g_{32}) = \delta g_{31} = 0$$
$$-\delta g_{03} = \chi.(g_{00} - g_{33}) = \delta g_{30} = -\chi.\{c^2.(1 - \frac{2.G.M}{c^2.r} - \frac{2.G.M}{c^4.r}) - 1\}$$
$$-\delta g_{10} = \chi.(g_{13} + g_{20}) = \delta g_{23} = 0$$
$$-\delta g_{11} = -\chi.(g_{12} - g_{21}) = \delta g_{22} = 0$$
$$-\delta g_{12} = -\chi.(g_{11} - g_{22}) = \delta g_{21} = 0$$
$$-\delta g_{20} = \chi.(g_{23} + g_{10}) = \delta g_{13} = 0$$

This is implying:

$$(11-b) : [G_{T-L}]$$
$$\downarrow$$
$$[G'_{T-L}]$$
$$=$$
$$[G_{T-L}]$$
$$\mp$$
$$\chi.\begin{bmatrix} 0 & 0 & 0 & c^2.(1 - \frac{2.G.M}{c^2.r} - \frac{2.G.M}{c^4.r}) - 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\{c^2.(1 - \frac{2.G.M}{c^2.r} - \frac{2.G.M}{c^4.r}) - 1\} & 0 & 0 & 0 \end{bmatrix}$$

and exhibiting an electrical-like field, also parallel to the axis:

$$(11-c) : E_x = \delta g_{01} = -H_x =, 0; E_y = \delta g_{02} = H_y = 0; H_z = -\delta g_{12} = 0$$
$$E_z = \delta g_{03} = \pm\chi.\{c^2.(1 - \frac{2.G.M}{c^2.r} - \frac{2.G.M}{c^4.r}) - 1\}$$

But this analysis is not yet achieved; it must now be fine tuned in considering the $\Phi$ matrix in detail; more precisely: “What is the value of the Christoffel's symbols of the second kind for an observer situated on the axis of rotation?”

The metric is not exactly standard but diagonal and symmetric. The relation (11-a) yields:

$$g = |g_{\mu\nu}| = c^2.(1 - X).(1 + X)^3; X = \frac{2.G.M}{c^2.r}$$

and when X $\neq$ 1:

$$g^{\mu\nu} = diag(\frac{1}{(c^2 - \frac{2.G.M}{r})}, \frac{1}{(1 + \frac{2.G.M}{c^2.r})}, \frac{1}{(1 + \frac{2.G.M}{c^2.r})}, \frac{1}{(1 + \frac{2.G.M}{c^2.r})})$$

In general within [[01]]:

$$\Gamma^{\sigma}_{\lambda\mu} = \frac{g^{\sigma\nu}}{2}.\{\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}}\}$$

I am only interested by the following symbols for $\lambda = 0$, 1, 2 and 3:

$$\Gamma^0_{\lambda 3} = \frac{g^{0\nu}}{2} . \{ \frac{\partial g_{3\nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda\nu}}{\partial x^3} - \frac{\partial g_{3\lambda}}{\partial x^\nu} \}$$

Since the metric and its inverse are diagonal, the quest can be restricted to:

$$\Gamma^0_{\lambda 3} = \frac{g^{00}}{2} . \{ \frac{\partial g_{30}}{\partial x^\lambda} + \frac{\partial g_{\lambda 0}}{\partial x^3} - \frac{\partial g_{3\lambda}}{\partial x^0} \}; \ \lambda = 0, 1, 2, 3.$$

As long as the observed position stays along the axis of rotation, the initial diagonal metrics stays diagonal and the off-diagonal terms are constantly null during the rotation; this implies that the non-vanishing Christoffel's symbols are:

$$\Gamma^0_{03} = \frac{g^{00}}{2} . \frac{\partial g_{00}}{\partial x^3}$$

$$\Gamma^0_{33} = -\frac{g^{00}}{2} . \frac{\partial g_{33}}{\partial x^0}$$

I now have to calculate:

$$\frac{\partial g_{00}}{\partial x^3} = -2 . G . M . \frac{\partial \frac{1}{r}}{\partial x^3} = 2 . G . M . \frac{x^3}{r^3} = -2 . G . M . \frac{z}{r^3}$$

$$\frac{\partial g_{33}}{\partial x^0} = \frac{2 . G . M}{c^2} . \frac{\partial \frac{1}{r}}{\partial x^0} = -\frac{2 . G . M}{c^2 . r^2} . \frac{\partial r}{\partial x^0}$$

It follows:

$$\Upsilon = \pm\chi = {\Gamma^0}_{\mu 3} . p^\mu = -\frac{g^{00} . G . M}{r^2} . \{ \frac{z . p^0}{r} - \frac{1}{c^2} . \frac{\partial r}{\partial x^0} . p_z \}$$

Provided **p**: ($p^0$, $p^1$, $p^2$, $p^3$) is equivalent to (m.c, 0, 0, 0) in the so-called zero approximation:

$$\Upsilon = \pm\chi = -\frac{c}{(c^2 - \frac{2 . G . M}{r})} . \frac{G . M . m}{r^3} . z$$

At the end of the day, if the TEQ would be true, there should be an electrical field along the z-axis :

$$E_z = \delta g_{03} = -c . \frac{1 - X - \frac{X+1}{c^2}}{1 - X} . \frac{G . M . m}{r^3} . z; \ X = \frac{2 . G . M}{c^2 . r}$$

The $\chi$ factor is proportional to the Newtonian gravitational force which is applied to a position along the z-axis (of rotation); the electrical field as well. This is -in some way- an electro-gravitational induced field. The X parameter is just the ratio between the Schwarzschild's radius of the Earth and the position on the z-axis. Concretely, this is at the North pole:

$$X(r = R_E) = \frac{2 . G . M}{c^2 . R_E} \sim \frac{2 . 6, 67 . 6}{9 . 6, 37} . \frac{10^{-11} . 10^{24}}{10^{16} . 10^6} \sim 1, 4 . 10^{-9}$$

Obviously X « 1, so that:

$$\frac{E_{North-Pole}(r = R_E)}{c} = \delta g_{03} \sim -\frac{G . M . m}{R_E^2}$$

**Conclusion**: "There is a coincidence between the EM-like field predicted by the TEQ along the axis of rotation of the Earth and the Newtonian gravitational field of the Earth." This is once more time a terrific duality.

## 2 Résumé

The GTR2 research proposal is an alternative theory. Its essential difference with the theory of relativity is the incorporation of the variations of the basis vectors until the second order. This choice allows the construction of a pseudo Riemann-Christoffel curvature tensor with the help of a so-called T-cube mimicking the Christoffel's cube and sometimes (but not systematically) identified with it. This is done to the cost of the appearance of structural (geometrical) EM-like fields. These strange fields can be related in a relatively simple manner to the components of the Riemann-Christoffel curvature tensor when the viewpoint of a Fermi's walker is adopted; for details, please see section 3(a). This is opening a discussion on a terrific duality between electromagnetic and gravitational fields, in general.

In this document, I have studied the complicated interplay between gravitation and EM fields in adopting another viewpoint. I have recalled a specific analysis of W. Heisenberg's uncertainty principle which I have proposed in section 3(b) because (i) that analysis is seemingly coherent with a fundamental result of GR (see figures 1 and 2) and (ii) furnishes a new generic expression for EM fields; i.e.: equation (2 ). In sub-section 1.5, I have then explained that, for a category of situations respecting the golden rule (equation (3 )), there is an equivalence between the EM fields obtained with the help of that unconventional analysis and the EM fields appearing in classical physics, for example: when the Lorentz-Einstein law is transcript into a differential equation with the language of operators; see section 3(h). At the end, this gave me the opportunity to confront this new formalism with E. Cartan's work scrutinizing the action of rotation on spinors or on bivectors; see end of book [[07]].

Although my analysis is not yet complete, I have started the study of situations for which the equivalence (terrific duality) between electromagnetic fields and variations of the metric might be plausible. Among them: the gravitons in subsection 1.10 and the Thirring-Lense effet around the Earth in subsection 1.11.

## 3 Personal contributions

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