History of the document: v1 (2003), document proposed for the IV^th Vigier Symposium in September 2003 in Paris, *not presented* but published in the Noetic journal. The initial document had a subtitle: Inertial motions in vacuum.) ; v2 (2007) only published on my homepage. The authorization for a free publishing of my own work (initially) has been asked to the journal and obtained. This authorization allows me now to present this document publicly. It is the third version, written and matriculated (French matriculation, given by the B.N.F.) in 2013.

Abstract:

With the help of an apparently trivial mathematical trick, we get a dynamic equation valid in vacuum. This ruse gives rise to a complete and sometimes complicated mathematical work called "The Theory of the (E) Question" based on the study of deformed (tensor, Lie) products. The dynamic equation describes a force of EM polarization per unit of volume. It exists if the EM field owns fluctuations. The quantum theory admits such variations around zero, it means in vacuum too. The exact nature of this force is a present question, but recent experiments made with high energy photonic lasers confirm that the light seems to be able to polarize spatial neutral regions in front of its trajectory. More interesting, this force should describe inertial motions in vacuum, i. e. some particles, although a Maxwell's vacuum contains neither a mass nor a charge. We know the experimental fact that a big enough nucleus owns an invariant volumetric mass density. We extend this fact to any particles considered between two interactions. This situation allows a special configuration for the force and the calculations of the authorized values for the volumetric mass densities. We discover only five families with two kinds of symmetry (in the mirror and triangle). We ask if it is a pure hazard or if we are in front of results coherent with one of the unification theories called SU(5).

1. Introduction:

Mathematical physics is a very active branch of the modern research area. Universities are developing ways of thinking with the hope to unify the different parts of a complicated puzzle called: the nature.

2. The classical background:

While one makes the discussion about a mass-less charged particle in an EM quantum vacuum [02], I begin the discussion in an apparently mass-less and charge-less EM quantum vacuum. This means in a part of space-time far away from any material source with or without charge. Maxwell's Laws [03; page 264] can be applied in such circumstances. The volumetric density of energy associated with an EM field in vacuum is:

(1)

$$\rho = (\varepsilon_0.E^2 + B^2/\mu_0) / 2$$

A partial derivation along the time yields:

(2)

$$\partial\rho/\partial t = (\varepsilon_0).\ \mathbf{E}.\ \partial\mathbf{E}/\partial t + (1/\mu_0).\ \mathbf{B}.\ \partial\mathbf{B}/\partial t$$

With the Maxwell's Laws, this is:

(3)

$$\partial\rho/\partial t = (1/\mu_0).\ [\mathbf{E.\ rot\ B} - \mathbf{B.\ rot\ E}]$$

The spatial part of the "classical" vacuum is supposed to be referred to a Euclidian 3-dimensional geometry. Consequently, classical calculations can be made and yield:

(4)

$$\partial\rho/\partial t = (1/\mu_0).\ \mathrm{div}[- (\mathbf{E} \wedge \mathbf{B}) - \mathbf{rot\ X}];\ \forall\mathbf{X}$$

Therefore, I always can define the following vector (which is a kind of Poynting's vector):

(5)

$$\mathbf{J} = (1/\mu_0).\ [(\mathbf{E} \wedge \mathbf{B}) + \mathbf{rot\ X}]];\ \forall\mathbf{X}$$

And write the following law of conservation:

(6)

$$\partial\rho/\partial t = -\ \mathrm{div}\ (\mathbf{J})$$

3. Existence of a dynamic equation in Maxwell's vacuum:

3.1. Demonstration:

I continue my demonstration with:

(7)

$$\partial \mathbf{J}/\partial t = (1/\mu_0).\ [(\partial \mathbf{E}/\partial t \wedge \mathbf{B}) + (\mathbf{E} \wedge \partial \mathbf{B}/\partial t) + \partial(\mathbf{rot\ X})/\partial t\ ];\ \forall \mathbf{X}$$

Here again I need the Maxwell's Laws and obtain:

(8)

$$\partial \mathbf{J}/\partial t = (1/\varepsilon_0.\mu_0^2).\ (\mathbf{rot\ B} \wedge \mathbf{B}) - (1/\mu_0).\ (\mathbf{E} \wedge \mathbf{rot\ E}) + (1/\mu_0).\ \partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

Before going further, please accept the following mathematical problematic that we shall later extend and call the "(E) question". Considering we could have to discuss about the possibility to write such equations:

(9)

$$|\mathbf{a} \wedge \mathbf{b}\rangle = [M].\ |\mathbf{b}\rangle + |\mathbf{k}\rangle$$

A very trivial solution to this apparently easy mathematical problem is the couple:

(10)

$$([M], \mathbf{k}) = (\Phi(\mathbf{a}), \mathbf{0}) \in M_3(\Re) \times (E_3, \Re)$$

With:

(11)

$$\Phi(\mathbf{a}) = \begin{bmatrix} 0 & -a^3 & a^2 \\ a^3 & 0 & -a^1 \\ -a^2 & a^1 & 0 \end{bmatrix}$$

Accept the definition of the following matrix (which is also an operator; see below § 5 - remark):

(12)

$$T_2(°)(\partial_x, \mathbf{a}) = \begin{bmatrix} \partial a^1/\partial x^1 & \partial a^1/\partial x^2 & \partial a^1/\partial x^3 \\ \partial a^2/\partial x^1 & \partial a^2/\partial x^2 & \partial a_2/\partial x^3 \\ \partial a^3/\partial x^1 & \partial a^3/\partial x^2 & \partial a^3/\partial x^3 \end{bmatrix}$$

Note that we have:

(13)

$$T_2(°)\ (\partial_x, \mathbf{a}) - T^t{}_2(°)(\partial_x, \mathbf{a}) = \Phi(\mathbf{rot\ a})$$

(14)

$$\mathrm{Tr}\ T_2(o)\ (\partial_x, \mathbf{a}) = \mathrm{div}\ (\mathbf{a})$$

Now, suppose we are exactly in the physical conditions permitting to write the trivial solution (10) for the EM field under consideration, starting with (8) yields:

(15)

$$\partial \mathbf{J}/\partial t = (1/\varepsilon_0.\ \mu_0^2).\ \Phi(\mathbf{rot\ B}).\ \mathbf{B} + (1/\mu_0).\ \Phi(\mathbf{rot\ E}).\ \mathbf{E} + (1/\mu_0).\ \partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

According to (11), develop (15) in:

(16)

$$\partial \mathbf{J}/\partial t = (1/\varepsilon_0.\mu_0^2).[T_2(°)(\partial_x, \mathbf{B}) - T^t{}_2(°)(\partial_x, \mathbf{B})].\ \mathbf{B} + (1/\mu_0).\ [T_2(°)(\partial_x, \mathbf{E}) - T^t{}_2(°)(\partial_x, \mathbf{E})].\ \mathbf{E} + (1/\mu_0).\partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

$$\downarrow$$

$$\partial(\varepsilon_0.\mu_0.\mathbf{J})/\partial t = (1/\mu_0).\ [T_2(°)(\partial_x, \mathbf{B}) - T^t{}_2(°)(\partial_x, \mathbf{B})].\ \mathbf{B} + (\varepsilon_0).\ [T_2(°)(\partial_x, \mathbf{E}) - T^t{}_2(°)(\partial_x, \mathbf{E})].\ \mathbf{E} + (\varepsilon_0).\partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

$$\downarrow$$

$$\partial(\varepsilon_0.\ \mu_0.\ \mathbf{J})/\partial t$$

$$=$$

$$[\varepsilon_0.\ T_2(o)(\partial_x, \mathbf{E}).\ \mathbf{E} + (1/\mu_0).\ T_2(o)(\partial_x, \mathbf{B}).\ \mathbf{B}] - [\varepsilon_0.\ T^t_2(o)(\partial_x, \mathbf{E}).\ \mathbf{E} + (1/\mu_0).\ T^t_2(o)(\partial_x, \mathbf{B}).\ \mathbf{B}] + \varepsilon_0.\ \partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

You may now verify that:

(17)

$$\mathbf{Grad_x}\ \rho = \varepsilon_0.\ T^t_2(o)(\partial_x, \mathbf{E}).\ \mathbf{E} + (1/\mu_0).\ T^t_2(o)(\partial_x, \mathbf{B}).\ \mathbf{B}$$

Demonstration.

Starting with $\rho = \frac{1}{2}.\ (\varepsilon_0.E^2 + B^2/\mu_0)$, a partial derivation along the different axis yields:

$$\partial\rho/\partial x^i = (\varepsilon_0).\mathbf{E}.\ \partial\mathbf{E}/\partial x^i + (1/\mu_0).\ \mathbf{B}.\ \partial\mathbf{B}/\partial x^i \text{ for } i = 1,2,3$$

Since we are working in a 3D Euclidian geometry, this is equivalent to:

$$\partial\rho/\partial x^i = (\varepsilon_0).\ E^j.\ \partial E^j/\partial x^i + (1/\mu_0).\ B^j.\ \partial B^j/\partial x^i \text{ for } i = 1, 2, 3$$

Recalling the possibility to write:

$$T_2(o)(\partial_x, \mathbf{E}) = \begin{bmatrix} \partial E^1/\partial x^1 & \partial E^1/\partial x^2 & \partial E^1/\partial x^3 \\ \partial E^2/\partial x^1 & \partial E^2/\partial x^2 & \partial E^2/\partial x^3 \\ \partial E^3/\partial x^1 & \partial E^3/\partial x^2 & \partial E^3/\partial x^3 \end{bmatrix}$$

$$T^t{}_2(o)(\partial_x, \mathbf{E}) = \begin{bmatrix} \partial E^1/\partial x^1 & \partial E^2/\partial x^1 & \partial E^3/\partial x^1 \\ \partial E^1/\partial x^2 & \partial E^2/\partial x^2 & \partial E^3/\partial x^2 \\ \partial E^1/\partial x^3 & \partial E^2/\partial x^3 & \partial E^3/\partial x^3 \end{bmatrix}$$

There is in fact no difficulty to verify that (17) holds.

We finally get:

(18)

$$\partial(\varepsilon_0.\mu_0.\ \mathbf{J})/\partial t = [(\varepsilon_0).\ T_2(o)(\partial_x, \mathbf{E}).\ \mathbf{E} + (1/\mu_0).\ T_2(o)(\partial_x, \mathbf{B}).\ \mathbf{B}] - \mathbf{Grad_x}\ \rho + (\varepsilon_0).\partial(\mathbf{rot\ X})/\partial t\ ;\ \forall \mathbf{X}$$

3.2. Interpretation:

Although one could be surprised by the complexity of (18) and not immediately recognize some already known forms, a complete analyse of it will reveal that it is a quite interesting equation. To get a better comprehension of this equation it is very important to remark that it owns the dimensions of a volumetric density of force.

Berkeley's University lectures and all the discussions about forces developed on an electrical (magnetic) polarized body by an external electrical (magnetic) field are well known [03; relation (9.22) page 308 and relation (10.19) page 369].

(19)

$$F_{\text{electric}}{}^i = \mathbf{m}^{\text{electric}}.\ \mathbf{Grad_x}\ E^i,\ i = 1, 2, 3$$

$$F_{\text{magnetic}}{}^i = \mathbf{m}^{\text{magnetic}}.\ \mathbf{Grad_x}\ B^i,\ i = 1, 2, 3$$

I propose to consider that if the vacuum can be polarized*, then the volumetric density of electrical and of magnetic momentum for the vacuum is given by:

(20)

$$\partial\mathbf{m}^{\text{electric}}/\partial\tau = (\varepsilon_0).\ \mathbf{E}$$

$$\partial\mathbf{m}^{\text{magnetic}}/\partial\tau = (1/\mu_0).\ \mathbf{B}$$

Therefore, I am persuaded of the existence, in these circumstances*, of an EM volumetric density of force coming from the polarization of the vacuum, given by:
(21)

$$\partial F_{\text{electric}}{}^{i} / \partial \tau = (\varepsilon_0).\ \mathbf{E}.\ \mathbf{grad}\ E^i,\ i = 1, 2, 3$$
$$\partial F_{\text{magnetic}}{}^{i} / \partial \tau = (1/\mu_0).\ \mathbf{B}.\ \mathbf{grad}\ B^i,\ i = 1, 2, 3$$

And we easily recognize:
(22)

$$\partial \mathbf{F}^{\text{ electric}} / \partial \tau = (\varepsilon_0).\ T_2(°)(\partial_\mathbf{x}, \mathbf{E}).\ \mathbf{E}$$
$$\partial \mathbf{F}^{\text{ magnetic}} / \partial \tau = (1/\mu_0).\ T_2(°)(\partial_\mathbf{x}, \mathbf{B}).\ \mathbf{B}$$

This is achieving to convince us that the two first terms of (18) represent the volumetric density of an EM polarization force in Maxwell's vacuum. Logically, other terms in (18) are another contribution to a volumetric density of force. It must be true for the term $\partial(\varepsilon_0.\ \mu_0.\ \mathbf{J})/\partial t$. This equation is a dynamic equation for this kind of vacuum and consequently tells the question if the variations along the time of the energetic flow (symbolized by the Poynting's vector **J**) generate inertial motions?

3.3. Discussion introducing the momentum conservation law:
In Einstein's theory the mass-energy equivalence relation results from mechanical and energetical considerations [01]. Even if the demonstration of (18) doesn't need any clear mechanical consideration, it can be supposed that all of these must be latent in the mathematical extra-tour called the (E) question. In modern physics, all forces are supposed to be related to a "carrier"; that is in fact a particular type of particles (the photon is carrying the EM interaction, the graviton is transporting the gravitational interaction and so and). Following this way of thinking must lead us to the conclusion that relation (18) implicitly contains information about the particles involved in an EM polarization of the vacuum. There is until now no difficulty to accept such kind of thoughts. Problems only arise as soon as one wants to exactly discover what kind of particles is represented in (18).

One possible way to surround the difficulty is based on a few numbers of remarks:
1°) All particles own a quasi-invariant volumetric density of energy during their existence; this is: ${}^{(3)}\rho$ = inv. [04; § 1.2.2. page 26]. Intuitively, this can only be true if the particle under consideration does not interact. This means if we study a given particle between two interactions: $\mathbf{Grad}_\mathbf{x}\ {}^{(3)}\rho = \mathbf{0}$.
2°) the choice for **X** is an arbitrary one. This allows to make the one for which $\partial(\mathbf{rot\ X})/\partial t = \mathbf{0}$.
3°) Believing to the existence of some beautiful symmetry in the nature, we look for circumstances for which the following relation could hold:
(23)

$$\chi_0.\ T_2(°)({}^{(3)}\partial_\mathbf{x}, {}^{(3)}\boldsymbol{\Gamma}).\ {}^{(3)}\boldsymbol{\Gamma} + \partial(\varepsilon_0.\ \mu_0.\ {}^{(3)}\mathbf{J})/\partial t = \mathbf{0}$$

The field ${}^{(3)}\boldsymbol{\Gamma}$ represents the spatial acceleration field for the particle under study. I thus suppose that local variations of the EM field can deform the geometry and induce a field of acceleration: $\boldsymbol{\Gamma}$. As we shall see later, this equation permits a comparison with the momentum conservation law. The first consequence of remarks 1°) 2°) and 3°) is the existence of a relation that could be understood as the affirmation that any small region in vacuum is an isolated one as soon as one incorporates the geometry into the play:
(24)

$$(\varepsilon_0).\ T_2(°)({}^{(3)}\partial_\mathbf{x}, {}^{(3)}\mathbf{E}).\ {}^{(3)}\mathbf{E} + (1/\mu_0).\ T_2(°)({}^{(3)}\partial_\mathbf{x}, {}^{(3)}\mathbf{B}).\ {}^{(3)}\mathbf{B} + (\chi_0).\ T_2(°)({}^{(3)}\partial_\mathbf{x}, {}^{(3)}\boldsymbol{\Gamma}).\ {}^{(3)}\boldsymbol{\Gamma} = \mathbf{0}$$

4. Extended products:

4.1. Justifications for the geometrical polarization:
Where could the term containing the acceleration come from? We call it the "geometrical polarization" and we shall justify this label in the next paragraphs. Since it should be, at least partially, related to a gravitational interaction, we should be able to connect them with the relativistic representation of this interaction, namely the extended "square" product $\triangle_{(\blacktriangledown\Gamma)}\ ({}^{(4)}\mathbf{u}, {}^{(4)}\mathbf{u})$ which is the extended product based on the Christoffel's cube of the proper 4D speed vector of the particle under study by itself. Mathematically, this can always be done. Indeed, in a first step, calculations yield in the case of an invariant Christoffel's cube:

(25)

$$d^2\triangle_{(\blacktriangledown\Gamma)}\,({}^{(4)}\mathbf{u},\,{}^{(4)}\mathbf{u})/d^2s\ \ - d\triangle_{(\blacktriangledown\Gamma)}\,({}^{(4)}\mathbf{u},\,{}^{(4)}\mathbf{u})/ds - c^2.\ \triangle_{(\blacktriangledown\Gamma)}\,({}^{(4)}\mathbf{\Gamma},\,{}^{(4)}\mathbf{\Gamma}) = \mathbf{0}$$

In a second step, let us consider some very common cosmological situations. For example: the Earth "turning" around the Sun under the effect of a central force. The trajectory of the planet is an ellipse. The more the dimensions of it are big and the more one will get the sensation that the central acceleration field is parallel transported by respect for itself in the local Levi-Civita connection. From this follows approximately that (see Annex § 4.2. below):

(26)

$$\triangle_{(\blacktriangledown\Gamma)}\,({}^{(4)}\mathbf{\Gamma},\,{}^{(4)}\mathbf{\Gamma}) \approx T_2(°)({}^{(4)}\partial_x,\,{}^{(4)}\mathbf{\Gamma}).\ {}^{(4)}\mathbf{\Gamma}$$

Although we obviously are considering circumstances characterized by (26) and by the resolution of (25) (obtained for an invariant connection), the new term $T_2(°)({}^{(4)}\partial_x,\,{}^{(4)}\mathbf{\Gamma}).\ {}^{(4)}\mathbf{\Gamma}$can effectively be connected to the variations of $\triangle_{(\blacktriangledown\Gamma)}\,({}^{(4)}\mathbf{u},\,{}^{(4)}\mathbf{u})$.

4.2. Annex: More on this geometrical polarization.

In this short section we shall study the parallel transport of an acceleration vector ${}^{(4)}\mathbf{\Gamma}$ along –what we could call a story inside- the phase space ${}^{(4)}\mathbf{u}(s)$; where ${}^{(4)}\mathbf{u}(s)$ represents in fact the 4D speed vector of some particle. This part of the work wants to be an extrapolation of the Appendix C in [12]. In accordance with the definition of a parallel transport, if the phase space is a 4D space vector (E_4, K) referred to a canonical basis $\Omega(\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, we have:

Cette courte section du travail est dédiée à l'étude du transport parallèle d'un vecteur accélération ${}^{(4)}\mathbf{\Gamma}$ dans l'espace des vitesses le long d'une trajectoire paramétrée; soit ${}^{(4)}\mathbf{u}(s)$ ce vecteur vitesse et « s » le paramètre retenu. Cette section se veut en fait une extrapolation de l'annexe C dans [12]. En accord avec la définition du transport parallèle y figurant et en supposant que l'espace des vitesses est un espace vectoriel de dimension 4 (E_4, K) référé à la base canonique $\Omega(\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, nous avons :

(4.2.1)

$$\forall\ \mu, \nu, \rho = 0, 1, 2, 3\ ;\ d\Gamma^\rho/ds + \Gamma_{\mu\nu}{}^\rho.\ du^\mu/ds.\ \Gamma^\nu = 0$$

As long as the following relation holds:

De sorte qu'aussi longtemps que la relation suivante est valide :

(4.2.2)

$$du^\mu/ds = \Gamma^\mu,$$

-Which is nothing else but the important assumption that the acceleration field under consideration "derives" from the variations of the speed field – Then:

-Ce qui ne signifie rien d'autre de très important que le champ d'accélération « dérive » du champ de vitesse- alors :

(4.2.3)

$$\forall\ \mu, \nu, \rho = 0, 1, 2, 3\ ;\ d\Gamma^\rho/ds + \Gamma_{\mu\nu}{}^\rho.\ \Gamma^\mu.\ \Gamma^\nu = 0$$

Within the approach involving extended products, this set of four relations can be summarized with:

Au sein de la présente approche (dédiée à l'introduction et l'usage de produits étendus en physique), les quatre relations précédentes peuvent encore s'écrire :

(4.2.4)

$$d^{(4)}\mathbf{\Gamma}/ds + \triangle_{(\blacktriangledown\Gamma)}({}^{(4)}\mathbf{\Gamma},\,{}^{(4)}\mathbf{\Gamma}) = \mathbf{0}.$$

If the following implicit relation holds:

Si nous pouvons écrire la relation implicite :

(4.2.5)

$${}^{(4)}\mathbf{\Gamma} = {}^{(4)}\mathbf{\Gamma}(\,{}^{(4)}\mathbf{u}),$$

The customary rules of derivation yields:

Les règles habituelles de la dérivation permettent de proposer aussi, dans un développement au premier ordre :

(4.2.6)

$$\forall\ \rho, \upsilon = 0, 1, 2, 3\ ;\ d\Gamma^{\rho}({}^{(4)}\mathbf{u})/ds = \partial\Gamma^{\rho}({}^{(4)}\mathbf{u})/\partial u^{\upsilon}.\ du^{\upsilon}/ds + 0(2)$$

With the help of relation (2) this is:

Avec la relation (2), ceci mène à :

(4.2.7)

$$\forall\rho, \upsilon = 0, 1, 2, 3\ ;\ d\Gamma^{\rho}({}^{(4)}\mathbf{u})/ds = \partial\Gamma^{\rho}({}^{(4)}\mathbf{u})/\partial u^{\upsilon}.\ \Gamma^{\upsilon}({}^{(4)}\mathbf{u}) + 0(2)$$

Of which a synthetic formulation exists:

Il existe une formulation synthétique de ce développement limité au premier ordre ; à savoir :

(4.2.8)

$$d^{(4)}\mathbf{\Gamma}({}^{(4)}\mathbf{u})/ds = T_2(°)(\partial_{\mathbf{u}}, \mathbf{\Gamma}({}^{(4)}\mathbf{u})).\ \mathbf{\Gamma}({}^{(4)}\mathbf{u}) + \mathbf{0}(2)$$

Finally, we get:

C'est ainsi que nous parvenons finalement à :

(4.2.9)

$$(4.2.2), (4.2.5), \triangle_{(\blacktriangledown\Gamma)}({}^{(4)}\mathbf{\Gamma}, {}^{(4)}\mathbf{\Gamma}) + T_2(°)(\partial_{\mathbf{u}}, \mathbf{\Gamma}({}^{(4)}\mathbf{u})).\ \mathbf{\Gamma}({}^{(4)}\mathbf{u}) + \mathbf{0}(2) = \mathbf{0}.$$

Formally, up to the terms of second order, this is the relation proposed in this document if the speed vector and the position vector are parallel: **u** // **x**. This is unfortunately in general not the case.
This configuration can be approximately obtained when the origin of the frame is chosen on the trajectory itself and for sufficiently short laps of time corresponding only to eventual negligible deviations of the trajectory.
One can accept to think that such approximate configuration can be realized for a particle "in gravitation" under the influence of a central force.

Aux termes de degré deux près, c'est la relation proposée si les vecteurs -vitesse et -position de la particule étudiée sont colinéaires : **u** // **x**. Ce n'est malheureusement en général pas le cas.
Cette configuration n'est obtenue par approximation qu'à la condition très restrictive de considérer a) l'origine des discussions et du référentiel où elles se font sur la trajectoire elle-même et b) cette trajectoire sur une période suffisamment courte pour que la déviation éventuelle qu'elle subit n'engendre qu'un angle négligeable entre le vecteur-position et le vecteur-vitesse de la particule.
On peut considérer que ce genre de situation approximative peut être réalisé pour une particule « gravitant » sous l'action d'une force centrale.

5. Streams in vacuum?

Let us come back to relation (23). It symbolically states equivalence between EM polarization forces and the pre-supposed representation of a kind of geometric polarization. Since we know that the Poynting's vector ${}^{(3)}\mathbf{J}$ represents an EM energetic flow and since we guess that the latter owns a speed ${}^{(3)}\mathbf{V}$, there is a great temptation to write the generic relation:

(27)

$${}^{(3)}\mathbf{J} = [\rho].\ {}^{(3)}\mathbf{V};\ [\rho] \in M_3(\Re)$$

Remark:

It is well-known that, if we write ${}^{(3)}\mathbf{\Gamma} = d^{(3)}\mathbf{u}(x, y, z, t)/dt$, then common rules of derivation are yielding:

(28)

$${}^{(3)}\mathbf{\Gamma} = T_2(°)\ ({}^{(3)}\partial_{\mathbf{x}}, {}^{(3)}\mathbf{u}).\ {}^{(3)}\mathbf{u} + \partial^{(3)}\mathbf{u}/\partial t$$

This relation should sound like an advertisement for the mathematician and suggest that matrices with the $T_2(°)({}^{(3)}\partial_{\mathbf{x}}, {}^{(3)}\ldots)$ formalism act like an operator of derivation (an ordinary one along the time) on $|{}^{(3)}\ldots >$. This remark can be in fact the starting point of a complete analyze on the subject.

Hypothesis:

We did make here a distinction between the speed **V** of the EM flow and the "geometric" speed **u** of the particle. This is an important point. Any swimmer trying to cross the river immediately recognizes the difference between the speed of the flow and its own speed. But if we suppose that the flow is always guiding the swimmer, then we do no longer have to make the difference: $\mathbf{V} \approx \mathbf{u}$. In that case (23) writes:

(29)

$$\chi_0 .\ c^2 .\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{\Gamma}).\ \{T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{u}).\ {}^{(3)}\mathbf{u} + \partial{}^{(3)}\mathbf{u}/\partial t\} + [\rho].\ \partial({}^{(3)}\mathbf{u})/\partial t + \partial[\rho]/\partial t.\ {}^{(3)}\mathbf{u} = \mathbf{0}$$

It can be re-organized:

(30)

$$\{\chi_0 .\ c^2 .\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{\Gamma}).\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{u}) + \partial[\rho]/\partial t\}.\ {}^{(3)}\mathbf{u} + \{\chi_0 .\ c^2 .\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{\Gamma}) + [\rho]\}.\ \partial({}^{(3)}\mathbf{u})/\partial t = \mathbf{0}$$

It holds independently on the spatial speed of the "thing" if:

(31)

$$\chi_0 .\ c^2 .\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{\Gamma}) + [\rho] = [0]$$

And:

(32)

$$\chi_0 .\ c^2 .\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{\Gamma}).\ T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{u}) + \partial[\rho]/\partial t = [0]$$

This is yielding:

(33)

$$\partial[\rho]/\partial t - T_2(°)({}^{(3)}\partial_x, {}^{(3)}\mathbf{u}).\ [\rho] = [0]$$

6. Eigenvalues of the volumetric density of equivalent-mass:

6.1. Characteristic equation:

To simplify, we write the volumetric density of equivalent mass:

(34)

$$\rho^* = (\rho/c^2)$$

And we calculate:

(35)

$$|\ \rho^* .\ I_3 + \chi_0 .\ T(°)(\partial_x, \mathbf{\Gamma})\ | = 0$$

This is in extenso:

(36)

$$\begin{vmatrix} \rho^* + \chi_0 .\ \partial\Gamma^1/\partial x^1 & \chi_0 .\ \partial\Gamma^1/\partial x^2 & \chi_0 .\ \partial\Gamma^1/\partial x^3 \\ \chi_0 .\ \partial\Gamma^2/\partial x^1 & \rho^* + \chi_0 .\ \partial\Gamma^2/\partial x^2 & \chi_0 .\ \partial\Gamma^2/\partial x^3 \\ \chi_0 .\ \partial\Gamma^3/\partial x^1 & \chi_0 .\ \partial\Gamma^3/\partial x^2 & \rho^* + \chi_0 .\ \partial\Gamma^3/\partial x^3 \end{vmatrix} = 0$$

It follows:

(37)

$$\begin{gathered} \rho^{*3} \\ + \\ (\chi_0).\ \rho^{*2}.(\partial\Gamma^1/\partial x^1 + \partial\Gamma^2/\partial x^2 + \partial\Gamma^3/\partial x^3) \\ + \\ (\chi_0)^2.\rho^*.[(\partial\Gamma^1/\partial x^1.\partial\Gamma^2/\partial x^2+\partial\Gamma^1/\partial x^1.\partial\Gamma^3/\partial x^3+\partial\Gamma^3/\partial x^3.\partial\Gamma^2/\partial x^2)-(\partial\Gamma^2/\partial x^1.\partial\Gamma^1/\partial x^2+\partial\Gamma^3/\partial x^1.\partial\Gamma^1/\partial x^3 + \partial\Gamma^3/\partial x^2.\partial\Gamma^2/\partial x^3)] \\ + \\ (\chi_0).\ ^3[\partial\Gamma^1/\partial x^1.\ \partial\Gamma^2/\partial x^2.\ \partial\Gamma^3/\partial x^3 - \partial\Gamma^1/\partial x^1.\ \partial\Gamma^3/\partial x^2.\partial\Gamma^2/\partial x^3 + \ldots] = 0 \end{gathered}$$

Let us also write:

(38)

$$\mathrm{div}\ \mathbf{\Gamma} = (\partial\Gamma^1/\partial x^1 + \partial\Gamma^2/\partial x^2 + \partial\Gamma^3/\partial x^3)$$

(39)

$$\begin{gathered} £ \\ = \end{gathered}$$

$(\partial\Gamma^1/\partial x^1 .\partial\Gamma^2/\partial x^2 + \partial\Gamma^1/\partial x^1 .\partial\Gamma^3/\partial x^3 + \partial\Gamma^3/\partial x^3 .\partial\Gamma^2/\partial x^2) - (\partial\Gamma^2/\partial x^1. \partial\Gamma^1/\partial x^2 + \partial\Gamma^3/\partial x^1. \partial\Gamma^1/\partial x^3 + \partial\Gamma^3/\partial x^2. \partial\Gamma^2/\partial x^3)]$

(40)

$$|T_2\,(°)(\partial, \mathbf{\Gamma})| = [\partial\Gamma^1/\partial x^1.\partial\Gamma^2/\partial x^2.\partial\Gamma^3/\partial x^3 - \partial\Gamma^1/\partial x^1.\partial\Gamma^3/\partial x^2.\partial\Gamma^2/\partial x^3 + ...]$$

We then obtain:
(41)

$$\rho^{*3} + (\chi_0).\ \mathrm{div}\ \mathbf{\Gamma}.\ \rho^{*2} + (\chi_0)^2.\ £.\ \rho^* + (\chi_0)^3.\ |\ T_2\,(°)(\partial, \mathbf{\Gamma})\ | = 0$$

6.2. Resolution of the characteristic equation:

6.2.1. Preliminarily remarks:

The equation (41) owns very interesting properties:

- i) It produces a kind of balance between the volumetric density of inertia, ρ^*, and the coefficient (χ_0) attached to the physical environment of the discussion (here the vacuum).
- ii) The coefficient in front of ρ^{*2} is the divergence of the acceleration field itself.
- iii) We must recall here that all fields of gravitation are supposed to be Newtonian fields and that they all are fields of acceleration. The converse is not certain; in extenso: if all fields of acceleration can effectively contribute to a phenomenon of gravitation (by the miracle of the equivalence principle), there is no certitude that the resulting effect remains Newtonian even if it is gravitational. **In the case of a coincidence between the acceleration field and the Newtonian gravitational field**, we should be able to write:

(42)

$$\mathrm{div}\ \mathbf{\Gamma} = -\ (\rho^*/\chi_0)$$

A consequence is that **the equation (41) falls into two parts**:

(43)

$$(\chi_0)^2.\ £.\ \rho^* + |(\chi_0).\ T_2\,(°)(\partial, \mathbf{\Gamma})| = 0$$

(44)

$$-\mathrm{div}\ \mathbf{\Gamma}.\ £ + |T_2\,(°)(\partial, \mathbf{\Gamma})| = 0$$

From which we deduce that, in that case, the determinant $|T_2\,(°)(\partial, \mathbf{\Gamma})|$ is proportional to the volumetric density ρ^* and to the divergence of the field.

- iv) Distinctions made in this theory between acceleration and Newtonian gravitation fields allow zero-divergence of the acceleration field with a non-zero virtual volumetric density of inertia. The immense advantage of my approach seems to lie in the possibility to separate the behaviour of the fields and of the potential sources. In fact, this way of thinking is relating experimental evidence: namely the omnipresence of empty regions, and a belief: all known particles should result from a kind of granulation (A little bit like the result of a crystallisation). For us, the matter appears simultaneously with the Newtonian gravitational field that is characterizing it. Thus, if the universe is a giant bath of energy resulting from an initial Big Bang, there must still exist some kinds of neutral energetic flows that are able to condense when the correct conditions are realized. This is an idea that can be found within some recent approaches [05].
- v) The complete resolution of this equation needs mathematical results found by Tartaglia and Cardan in the century 16th [06].

6.2.2 Preliminaries of the resolution:

Following the method proposed by Tartaglia-Cardan, we introduce the variable:
(45)

$$z = \rho^* + 1/3\ (\chi_0.\ \mathrm{div}\ \mathbf{\Gamma})$$

We get:
(46)

$$z^3 + [-\ 1/3\ (\chi_0.\ \mathrm{div}\ \mathbf{\Gamma})^2 + (\chi_0{}^2\ £)].\ z + [2/27\ (\chi_0.\ \mathrm{div}\ \mathbf{\Gamma})^3 - 1/3\ (\chi_0{}^3.\ £.\ \mathrm{div}\ \mathbf{\Gamma}) + |\ (\chi_0).\ T_2(°)(\partial, \mathbf{\Gamma})\ |\ ] = 0$$

Therefore, we write:

(47)

$$z = z_1 + z_2$$

(48)

$$p = [- 1/3\ (\chi_0.\ \mathrm{div}\ \mathbf{\Gamma})^2 + (\chi_0^2.\ £)]$$

(49)

$$q = [2/27\ (\chi_0.\ \mathrm{div}\ \mathbf{\Gamma})^3 - 1/3\ (\chi_0^3.\ £.\ \mathrm{div}\ \mathbf{\Gamma})) + |\chi_0.\ T_2(°)(\partial,\mathbf{\Gamma})|\ ]$$

(50)

$$\Delta = q^2 + (4/27).\ p^3$$

6.2.3. Generalities on the solutions:
When coordinates of the acceleration field $\mathbf{\Gamma}$ are taking all different possible real values, we obtain several types of situations classified in:
(51)

$$\Delta > 0 \text{ results in one real value given by: } z = {}^3\sqrt{[-q/2 - ({}^2\sqrt{\Delta})/2]} + {}^3\sqrt{[-q/2 + ({}^2\sqrt{\Delta})/2]}$$

(52)

$$\Delta = 0 \text{ results in one real value given by: } z = 2\ {}^3\sqrt{(-q/2)}$$

(53, 54 and 55)

$$\Delta < 0 \text{ results in three real values}$$

To our surprise, when Δ is describing all real numbers, **we find out only 5 types of solutions**. I classify this result of my theory as a strange hazard, but I ask if it is reasonable to try a comparison with one of the proposed theories for unification of the world of the particles: SU(5)(quark downright red, green, and blue; e^+ right; ν right) [07].

6.2.4. Some remarks for a graphical resolution of the equations:
It can be demonstrated that equations above form a system of the two equations mathematically equivalent to:
(56)

$$2x.\ (x^2 - 3y^2) = A$$

(57)

$$2y.\ (3x^2 - y^2) = B$$

This system offers some interesting symmetries:
Once more we should be surprised by strange symmetries appearing in this system of equations:
1°) if one tries to inverse x and y then one gets a similar system of equations but with other coefficients; what I could call the mirror-symmetry (**right hand - left hand; or matter and anti-matter?)**. We are in fact in front of a unique family of curves defined by $x.\ (x^2 - 3y^2) = f(A/2)$. To find out the solutions of the system is at the end equivalent to the study of the intersection between $f(A/2)$ and $f^{-1}(-B/2)$ under two conditions:
(58)

$$B > 0 \text{ and } A \in]\ -2.|z|^3, +2|z|^3[$$

2°) if (x, y) is a solution of this system, then couples obtained after a rotation of $2\pi/3$ et $4\pi/3$ are solutions too, my so-called triangle-symmetry (**the trinity of the quarks or of the neutrinos?**)

6.3. If we could write $\Gamma = G$ (Newton):
What would happen to this model if the acceleration field could be exactly compared with the acceleration field induced by a Newtonian source in some region of space-time faraway from this source (*weak gravitation field*)? Spatial coordinates of a Newtonian acceleration field (the well-known gravitation field) are given by [01]:
(59)

$$\Gamma^i = G.\ M.\ (x^i / R^3)$$

The mass M is the source, G is the gravitational constant, R is the distance between the source and the place of the measure. So that we get:
(60)

$$\partial\Gamma^i/\partial x^j = (G.\ M / R^3).\ [\delta^i_j - 3.\ x^i.\ x^j / R^2]$$

This is inducing:
(61)

$$\text{div}\ \mathbf{\Gamma} = 0$$

That is the nullity of the divergence of the Newtonian acceleration field induced by the source M. Note that it is always true if it is measured outside from the source.

(62)

$$£ = -3.\ (G.\ M / R^3)^2$$

(63)

$$|T_2\,(°)(\partial, \mathbf{\Gamma})| = 2.(G.\ M / R^3)^3$$

Remember that the equation to be resolved is given by (41). The above results (61), (62) and (63) finally yield:
(64)

$$\rho^{*3} - 3(\chi_0)^2\ .(G.M/R^3)^2.\rho^* + 2(\chi_0)^3(G.M/R^3)^3 = 0$$

This can be written in a simpler form:
(65)

$$[\rho^* - (\chi_0).\ G.\ M/R^3)]^2.[\rho^* + 2.\ (\chi_0).\ G.\ M/R^3)] = 0$$

From which we deduce that Newtonian weak acceleration fields always have two types of associated eigenvalues:
(66)

A double one given by: $(\chi_0.\ G.\ M/R^3)$

(67)

A simple one given by $-2(\chi_0.\ G.\ M/R^3)$

Studying the eigen-vectors associated to these eigen-values will give us later the acceptable physical values for the volumetric densities of inertial equivalent masses in such a vacuum, depending on the sign of the yet unknown constant χ_0.

7. Temporary conclusion; possible connections with other models?

Just suppose that the source M could be uniformly dispersed in vacuum over the distance R around a central point. We would have everywhere a volumetric density of matter "ρ" (and equivalent, of energy) given by:
(68)

$$M = (4\pi.\ R^3).\ (\text{"}\rho\text{"}/3)$$

Now, consider the solution (67) and get:
(69)

$$\rho^*\ (\text{solution of 41}) = -\ (8.\ \chi_0.\ G.\ \pi.\ \text{"}\rho\text{"})/3$$

The energetic part of the dynamic equation obtained from the generalized theory of relativity in an isotropic and homogeneous model related to the Robertson-Walker metric is: $(8.\ G.\ \pi.\ \text{"}\rho\text{"})/3$ [08]. To realize the coherence of this model, we should absolutely introduce the relation:
(70)

$$\rho\ (\text{Solution of 41}) = \text{"}\rho\text{"}$$

It gives us the value of the constant χ_0:

(71)

$$-8.\ \chi_0.\ G.\ \pi/3 = 1$$

The resolution of: $(\rho/\chi_0)^3 + \text{div}\,\mathbf{\Gamma}.\ (\rho/\chi_0)^2 + £.\ (\rho/\chi_0) + |T_2\,(°)(\partial,\mathbf{\Gamma})| = 0$ will give us the different possible values of the quantity (8.G.π"ρ")/3. This quantity has several interpretations depending on the context within we decided to make the discussions. In a classical context it is the volumetric density of potential energy. In a relativistic context it is the energetic part of a dynamic equation giving the relation between the geometry of the space-time and the distribution of the energy in this space-time. According to the experimental fact that our universe seems to be quite everywhere spatially flat (*with two important exceptions: around elementary particles and near big concentration of mass*), we should get the logical conclusion that our universe only satisfies the virial theorem or owns a temporal curvature given by: $(dR/dt)^2/R^2 = (8.G.\pi"\rho")/3$ (Relativistic frame).

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